Technical Note R-101

Numerical Analysis of Plug Nozzles by the Method of Characteristics

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HUNTSVILLE, ALABAMA

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NUMERICAL ANALYSIS OF PLUG NOZZLES BY THE METHOD OF CHARACTERISTICS

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ABSTRACT

This report describes the theory used to calculate supersonic flow in plug nozzles and the computer program based on this theory. Flow properties are calculated by the method of characteristics. Sauer's transonic theory is used to determine the starting line and Korst's technique is used to calculate the base pressure.

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LIST OF SYMBOLS

A	Area, ft^2
a	Sound speed, ft/sec
C_2	Crocco number, defined in Equation 25
M	Mach number
m	Mass flow rate, lbm/sec
M_o	Momentum flux, lbf
M_p	Pressure thrust, lbf
M_{red}	Reduced Mach number, defined in Equation 36
P	Pressure, lbf/ft^2
P_b	Base pressure, lbf/ft^2
RT	Radius from the throat to origin, ft
s	Entropy, ft^2/sec^2, °R
T	Temperature, °R
V	Velocity
$\tilde{u}, \tilde{v}$	Velocity components in $\bar{x}$, $\bar{y}$ direction
x, y	Cartesian coordinates, ft
$\bar{x}, \bar{y}$	Reference coordinates as shown in Figure 4, ft
y_s	Radius of nozzle throat, ft

Greek Symbols

α	Defined in Equation 24, ft^{-1}
β	Mach angle, $\sin^{-1}\left(\frac{1}{M}\right)$

γ	Ratio of specific heats
θ	Flow angle
ν	Prandtl-Meyer expansion turning angle
ρ	Density, lbm/ft^3
ρ_s	Radius of curvature at the wall of nozzle throat, ft
ϕ^*	Throat plane inclined angle

INTRODUCTION

During the past few years many research groups have engaged in study of the performance characteristics of a plug nozzle. As yet a computer program to study the flow pattern and performance has not been reported. This report summarizes a basic analytical method and describes a computer program based on this method.

The basic characteristic equations were derived by assuming rotational flow, so that, in the future, shock equations could be added to the present calculations without difficulty. The gas is assumed to be perfect and inviscid. Friction loss on the nozzle wall is ignored, and the base pressure of the plug is computed by using Korst's theory.

The present numerical method has been programmed in IBM 7040 FORTRAN IV, and two sample calculations are presented in this report. This program can be used to examine the performance of various plug nozzle design concepts.

ANALYSIS

The flow field of a plug nozzle is formed by an axisymmetric internal plug with an external solid boundary at the upstream and free expansion at the downstream. It consists of a base pressure region at the end of the plug if the plug is truncated.

The method of characteristics is used to calculate the supersonic flow fields and the Prandtl-Meyer relations are used to calculate the flow properties of the lip of shroud. The base pressure problem is soived by using Korst's theory. The gas is assumed to be perfect, inviscid, and the flow field is assumed to be steady, rotational and axisymmetric.

Basic Equations of the Method of Characteristics

The characteristic equations for axisymmetric, steady and rotational flow used in this analysis were presented by A. H. Shapiro in Reference 1. The characteristic equations were derived from continuity, energy and Euler's equations. The detailed derivations were also shown in Reference 2. There are two families of characteristics:

Left Running Characteristic

$$\cot \beta \frac{dV}{V} - d\theta - \frac{\sin \beta \sin \theta}{y \cos (\beta + \theta)} dx + \frac{T}{a^2} \sin \beta \cos \beta \, ds = 0 \quad . \tag{1}$$

Right Running Characteristic

$$\cot \beta \frac{dV}{V} + d\theta - \frac{\sin \beta \sin \theta}{y \cos (\theta - \beta)} dx + \frac{T}{a^2} \sin \beta \cos \beta \, ds = 0 \quad . \tag{2}$$

The geometric properties of the characteristics provide other relations:

Left Running Characteristic

$$\frac{dy}{dx} = \tan(\theta + \beta) \tag{3}$$

Right Running Characteristic

$$\frac{dy}{dx} = \tan(\theta - \beta) \tag{4}$$

Writing Equations 3 and 4 in finite difference form and solving for x, y, one obtains:

$$x_3 = \frac{x_1 + \dfrac{1}{[\tan(\theta + \beta)]_{13}} \left\{y_2 - y_1 - x_2 [\tan(\theta - \beta)]_{23}\right\}}{1 - \dfrac{[\tan(\theta - \beta)]_{23}}{[\tan(\theta + \beta)]_{13}}} \tag{5}$$

and

$$y_3 = y_2 + [\tan(\theta - \beta)]_{23} (x_3 - x_2) \quad . \tag{6}$$

The last terms in Equations 1 and 2 are to take into account the entropy change in the flow field. In order to compute the entropy change along a characteristic, the entropy is assumed to be constant along a streamline and varied across a streamline. Since the entropy gradient is not large, it is also assumed to be constant in each small region. The derivations were presented in Reference 1 and the expression can be written as follows:

$$s_3 = s_1 + \frac{(s_2 - s_1)(x_3 - x_1)\left[\frac{\sin\beta}{\cos(\theta+\beta)}\right]_{13}}{(x_3 - x_1)\left[\frac{\sin\beta}{\cos(\theta+\beta)}\right]_{13} + (x_3 - x_2)\left[\frac{\sin\beta}{\cos(\theta-\beta)}\right]_{23}} \quad . \tag{7}$$

The velocity and the flow angle at point 3 can be solved by combining Equations 1 and 2:

$$V_3 = \frac{1}{\left[\frac{\cot\beta}{V}\right]_{13} + \left[\frac{\cot\beta}{V}\right]_{23}} \Bigg\{ \theta_2 - \theta_1 + \left(\frac{\cot\beta}{V}\right)_{13} V_1 + \left(\frac{\cot\beta}{V}\right)_{23} V_2$$

$$+ \left[\frac{\sin\beta\sin\theta}{y\cos(\theta+\beta)}\right]_{13}(x_3 - x_1) + \left[\frac{\sin\beta\sin\theta}{y\cos(\theta-\beta)}\right]_{23}(x_3 - x_2) \tag{8}$$

$$- \left[\frac{T}{a^2}\sin\beta\cos\beta\right]_{13}(s_3 - s_1) - \left[\frac{T}{a^2}\sin\beta\cos\beta\right]_{23}(s_3 - s_2) \Bigg\}$$

and

$$\theta_3 = \theta_1 + (V_3 - V_1)\left[\frac{\cot\beta}{V}\right]_{13} - \left[\frac{\sin\beta\sin\theta}{y\cos(\theta+\beta)}\right]_{13}(x_3 - x_1)$$

$$+ \left[\frac{T}{a^2}\sin\beta\cos\beta\right]_{13}(s_3 - s_1) \quad . \tag{9}$$

When a right characteristic intersects the boundary, as shown in Figure 2, the intersection can be solved by the following equations:

$$y_B = y_{B1} + (x_B - x_{B1})\tan\theta_B \tag{10}$$

$$y_B = y_1 + (x_B - x_1)\left[\tan(\theta - \beta)\right]_{1,B} \tag{11}$$

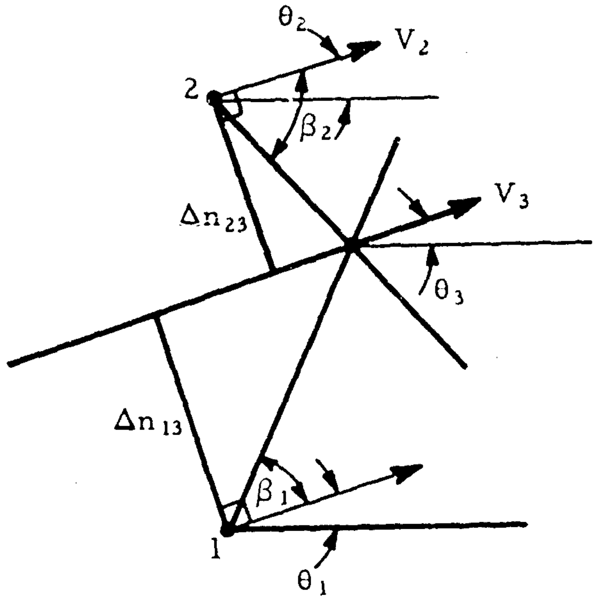

Figure 1. Nomenclature for Method of Characteristics in Rotational Flow Field Calculations

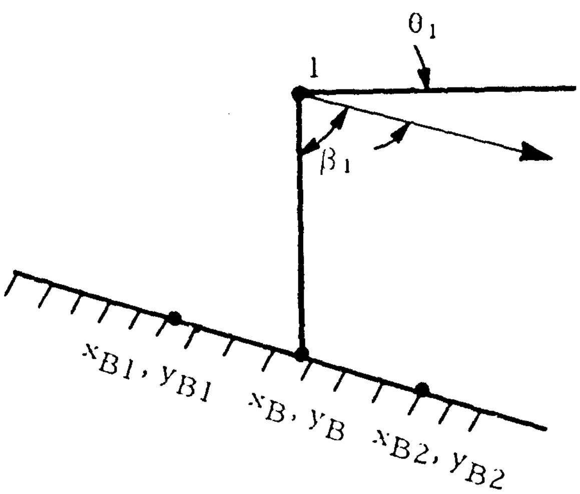

Figure 2. Nomenclature for Method of Characteristics in Boundary Point Calculation

where

$$\tan \theta_B = \frac{y_{B2} - y_{B1}}{x_{B2} - x_{B1}} \quad .$$

Equating Equations 10 and 11, one obtains

$$x_B = \frac{y_1 - y_{B1} - x_1 \left[\tan(\theta - \beta)\right]_{1B} + x_{B1} \tan \theta_B}{\tan \theta_B - \left[\tan(\theta - \beta)\right]_{1B}} \quad . \tag{12}$$

The entropy along the boundary is assumed to be constant throughout the flow field, and the velocity on the boundary can be computed by using Equation 2.

$$V_B = V_1 + \left[V \tan \beta\right]_{1B} \left\{ \theta_1 - \theta_B + \left[\frac{\sin \theta \sin \beta}{y \cos(\theta - \beta)}\right]_{1B} (x_B - x_1) - \left[\frac{T}{a^2} \sin \beta \cos \beta\right]_{1B} (s_B - s_1) \right\} \quad . \tag{13}$$

When a left running characteristic intersects the boundary by using a similar method as shown above the following equations can be obtained:

$$x_B = \frac{y_1 - y_{B1} - x_1 \left[\tan(\theta + \beta)\right]_{1B} + x_{B1} \tan \theta_B}{\tan \theta_B - \left[\tan(\theta + \beta)\right]_{1B}} \tag{14}$$

$$y_B = y_{B1} + (x_B - x_{B1}) \tan \theta_B \tag{15}$$

$$V_B = V_1 + \left[V \tan \beta\right]_{1B} \left\{ \theta_B - \theta_1 + \left[\frac{\sin \beta \sin \theta}{y \cos(\theta + \beta)}\right]_{1B} (x_B - x_1) - \left[\frac{T}{a^2} \sin \beta \cos \beta\right]_{1B} (s_B - s_1) \right\} \tag{16}$$

In order to compute the flow properties at the end point of the boundary, it is necessary to insert a characteristic at that point as shown in Figure 3. When a left running characteristic is inserted, the intersection may be solved by using the following equations:

$$[\tan(\theta + \beta)]_{34} = \frac{y_4 - y_3}{x_4 - x_3} \tag{17}$$

$$\frac{y_4 - y_1}{x_4 - x_1} = \frac{y_2 - y_1}{x_2 - x_1} \quad . \tag{18}$$

Solving x_4 from Equations 17 and 18 yields

$$x_4 = \frac{y_1 - x_3 + x_3[\tan(\theta + \beta)]_{34} - x_1\left(\frac{y_2 - y_1}{x_2 - x_1}\right)}{[\tan(\theta + \beta)]_{34} - \frac{y_2 - y_1}{x_2 - x_1}} \quad . \tag{19}$$

The velocity at point 3 can be computed by using the following equation:

$$\begin{aligned} V_3 = V_4 + [V \tan\beta]_{34} \Bigg\{ & (\theta_3 - \theta_4) + \left[\frac{\sin\beta \sin\theta}{y \cos(\theta + \beta)}\right]_{34} (x_3 - x_4) \\ & - \left[\frac{T}{a^2} \sin\beta \cos\beta\right]_{34} (s_3 - s_4) \Bigg\} \quad . \end{aligned} \tag{20}$$

Similarly, the relation for a right running inserted characteristic can be written as follows:

$$x_4 = \frac{y_1 - y_3 + x_3[\tan(\theta - \beta)]_{34} - x_1\left(\frac{y_2 - y_1}{x_2 - x_1}\right)}{[\tan(\theta - \beta)]_{34} - \frac{y_2 - y_1}{x_2 - x_1}} \tag{21}$$

$$V_3 = V_4 + [V \tan\beta]_{34} \left\{ (\theta_4 - \theta_3) + \left[\frac{\sin\beta \sin\theta}{y \cos(\theta - \beta)}\right]_{34} (x_3 - x_4) - \left[\frac{T}{a^2} \sin\beta \cos\beta\right]_{34} (s_3 - s_4) \right\} .$$

Transonic Region

The transonic flow near the throat of a nozzle requires special treatment because the method of characteristics is not valid in this region. The Sauer analysis in Reference 3 offers a solution to this problem. The solution was presented as a power series and the derivation was based on two dimensional, small perturbations theory.

$$\tilde{u} = \alpha\bar{x} + \frac{\gamma+1}{2} \alpha^2 \bar{y}^2 + \dots \tag{22}$$

$$\tilde{v} = (\gamma+1) \alpha^2 \bar{x}\bar{y} + \frac{(\gamma+1)^2}{6} \alpha^3 \bar{y}^3 + \dots \tag{23}$$

where

$$\alpha = \sqrt{\frac{1}{(\gamma+1)\rho_s y_s}} . \tag{24}$$

The values ρ_s and y_s can be obtained from the geometry of a nozzle as shown in Figure 4.

Base Pressure Region

When a plug nozzle is truncated, the base pressure becomes an important parameter affecting the nozzle performance. Korst's analysis in Reference 4 provides an approach to this problem. The derivations are

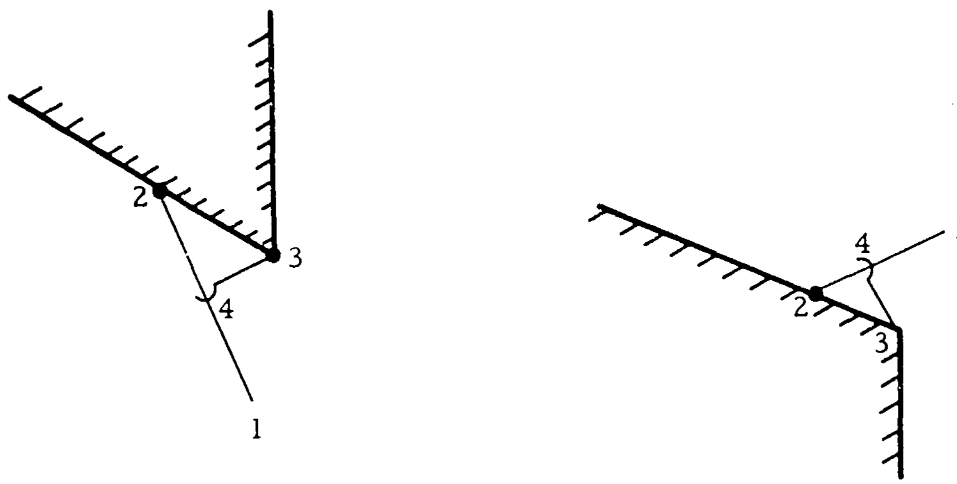

Figure 3. Nomenclature for an Inserted Characteristic

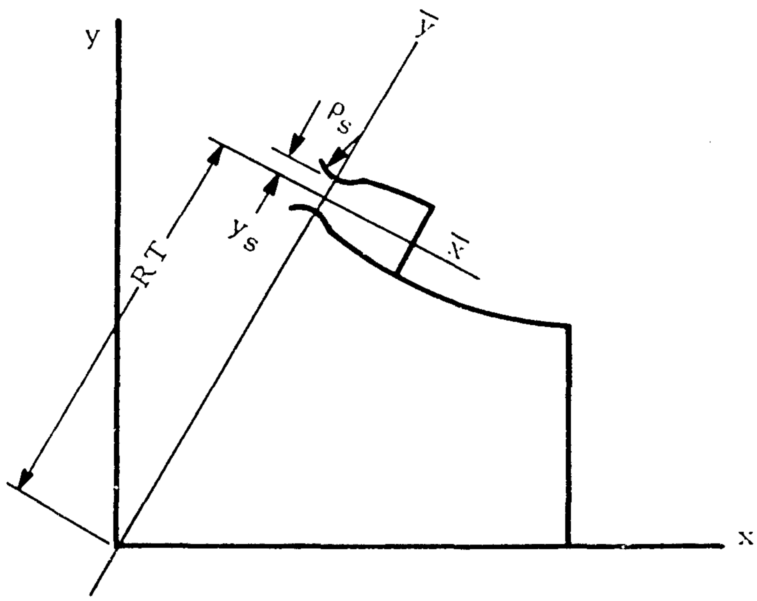

Figure 4. Internal-External Expansion Plug Nozzle Configuration

based on two dimensional turbulent flow with constant pressure mixing. The essential feature of the flow model is shown in Figure 5. The boundary layer at separation is assumed to be thin compared to the length of the jet mixing region and no mass is assumed to bleed into the wake.

The Crocco number is defined as follows:

$$C_2 = \sqrt{\frac{M_2^2}{\frac{2}{\gamma+1} + M_2^2}} \quad . \tag{25}$$

For isoenergetic, fully-developed, turbulent, constant pressure, jet mixing profiles, the velocity ratio is

$$\psi_j = \frac{u}{u_2} = \frac{1}{2}(1 + \mathrm{erf}\,\eta) \tag{26}$$

where

$$\mathrm{erf}\,\eta = \frac{2}{\sqrt{\pi}} \int_0^{\eta} e^{-\beta^2}\, d\beta \tag{27}$$

$$\eta = \sigma \frac{y}{x} \tag{28}$$

and

$$\sigma = 12 + 2.758\, M_2 \quad . \tag{29}$$

In the case of no-bleed into the wake, the Crocco number at j streamline is

$$C_d^2 = \psi_j^2\, C_2^2 \quad . \tag{30}$$

Assuming $M_{2a} = M_{3a}$, the isentropic relation gives

$$\frac{P_4}{P_3} = \left(\frac{P_0}{P}\right)_{2d} = \frac{1}{(1 - C_d{}^2)^{\frac{\gamma}{\gamma - 1}}} \quad . \tag{31}$$

The flow turning angle ${}_1\theta_2$ can be obtained from the Prandtl-Meyer relation by assuming ${}_1\theta_2 = {}_3\theta_4$, and the Mach number at region 1 can also be obtained from the Prandtl Meyer relation:

$$M_1 = f\ (\nu_1) \tag{32}$$

where

$$\nu_1 = \nu_2 - {}_1\theta_2 \quad . \tag{33}$$

Using isentropic relations, the base pressure for back step can be computed as follows:

$$\frac{P_1}{P_b} = \left.\frac{P_1}{P_0}\right|_{M_1} \times \left.\frac{P_0}{P_2}\right|_{M_2} \quad . \tag{34}$$

By assuming a series of values for M_2 in Equation 25, and carrying through the whole procedure, a curve, $\frac{P_b}{P_1}$ vs M_1 can be obtained.

In order to take into account the effect of boattailing, Korst's Reduced Mach Number Concept has to be used to extend the previous technique. $\theta_a{}^*$ was defined as a streamline angle at which M = 1 produced by the Prandtl-Meyer relation from M_{1a} and θ_{1a} .

$$M_{1a} = M_{1a}\ (-\theta_{1a} - \theta_a{}^*) \tag{35}$$

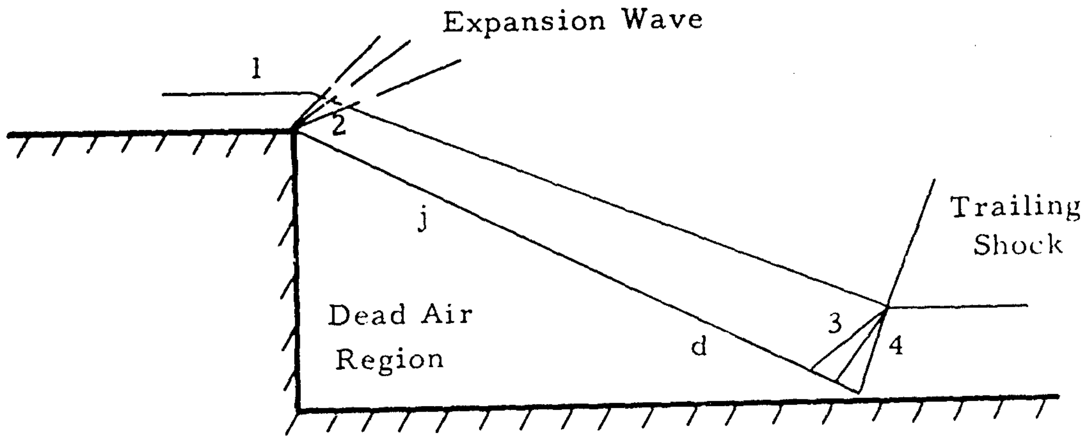

Figure 5. Korst's Flow Model

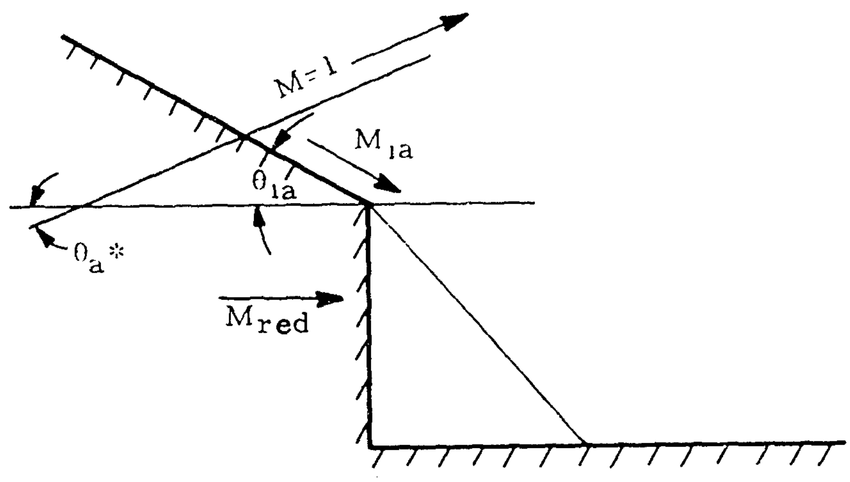

Figure 6. Geometrical Configuration of Base Region

A reduced Mach number can be determined as follows:

$$M_{red} = M_{red}\,(-\,\theta_a^{*}) \quad . \tag{36}$$

$\frac{P_b}{P_1}$ vs M_1 curve obtained from the previous technique is taken as $\frac{P_2}{P_{red}}$ vs M_{red}. Then, the base pressure can be computed by using the following relations:

$$\frac{P_b}{P_1} = \frac{P_2}{P_{red}} \cdot \frac{P_{0a}}{P_1} \cdot \frac{P_{red}}{P_{0a}} \tag{37}$$

where

$$\frac{P_1}{P_{0a}} = \frac{P_1}{P_{0a}} \quad (M_1) \tag{38}$$

$$\frac{P_{red}}{P_{0a}} = \frac{P_{red}}{P_{0a}} \quad (M_{red}) \quad . \tag{39}$$

<u>Numerical Procedure</u>

The computations consist of several distinct parts: the calculations of a starting line, field points and boundary points, Prandtl-Meyer expansion, and the base pressure. The starting line is determined by using Equations 22, 23 and 24. The computation of field points and boundary points is performed by a regular iteration scheme. The coefficients of mean values are employed in the process as suggested by Darwell in Reference 5. The calculation of Prandtl-Meyer expansion takes part in the process when the last upper boundary point is obtained. When the last point of the lower boundary is reached, the base pressure computation is employed.

The cumulative vacuum thrust is made up of the momentum flux and the pressure thrust at the starting line plus the pressure integral on the boundaries. The mass flow rate across the segment $\overline{12}$ as shown in Figure 7 is

$$m = \rho_{12} V_{12} A_{12} \cos (\phi - \theta_{12}) \tag{40}$$

where

$$\phi = \tan^{-1} \left(\frac{x_1 - x_2}{y_2 - y_1}\right) \tag{41}$$

and

$$A_{12} = \pi (y_1 + y_2) \sqrt{(x_2 - x_1)^2 + (y_1 - y_2)^2} \quad . \tag{42}$$

The momentum flux and pressure thrust at the segment $\overline{12}$ at the starting line are

$$M_o = \frac{m}{g} V_{12} \cos \theta_{12} + P_{12} A_{12} \cos \phi \quad . \tag{43}$$

The pressure integral at segment $\overline{12}$ on the plug is

$$M_p = P_{12} A_{12} \cos \phi \quad . \tag{44}$$

The cumulative vacuum thrust can be computed as follows:

$$T = \sum_{(S)} M_o + \sum_{(B)} M_p \quad . \tag{45}$$

The vacuum thrust coefficient is defined as follows:

$$(C_F)_{vac} = \frac{T + P_b \pi r_D^2}{P_0 A_T} \quad . \tag{46}$$

The numerical procedure described in this report has been programmed in IBM 7040 computer FORTRAN IV language.

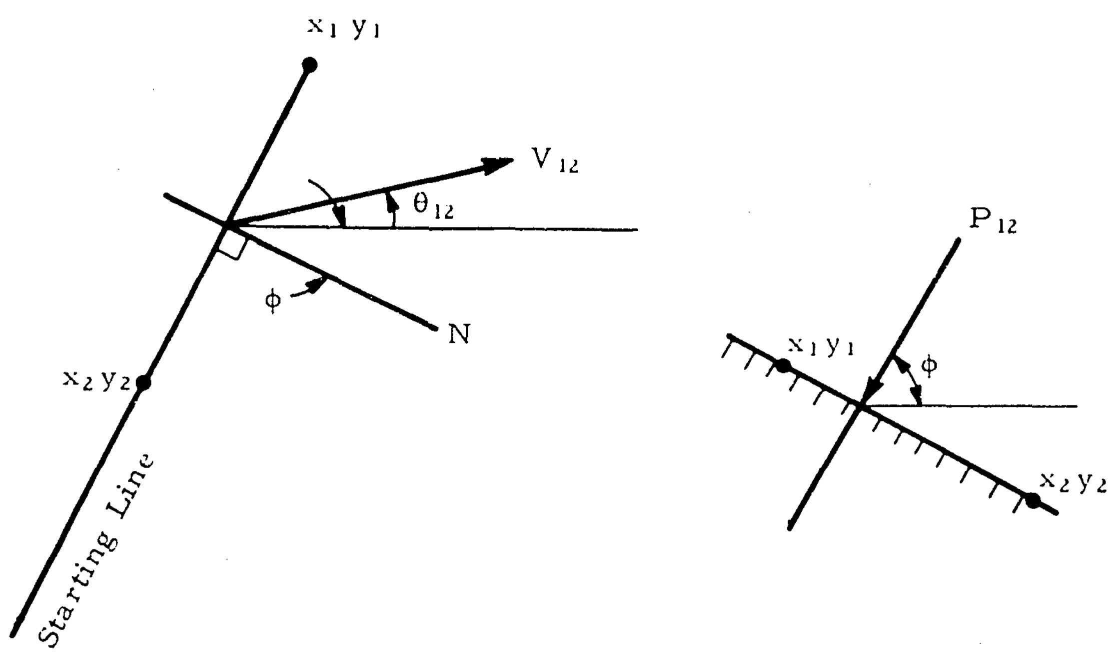

Figure 7. Illustration of Definition of Thrust Calculation

REMARKS ON CALCULATIONS

The accuracy of the present method depends on the net size chosen for the calculations. In other words, the smaller the net size one chooses, the more accurate the results one can obtain. When the small net size is used, of course, the points used to describe the contour should be more accurate. There are two ways to control the net size. One is to control the number of points at the starting line, and the other is to control the number of rays at the lip of a nozzle.

When the inclined angle of the lower wall becomes large, the reduced Mach number computed from Equation 36 differs from the Mach number at the edge in a great amount. This difference may cause the base pressure to be greater than the pressure on the boattailed portion as shown in Equation 37. In this case, it may indicate separation and the theory becomes invalid.

SAMPLE RESULTS AND DISCUSSION

The program has been used to compute several test cases. Two typical cases are selected for presentation in this report. An extêrnal expansion plug nozzle was designed by using the program in Reference 6. In order to compute a starting line for the analysis, the simple wave relation was employed. The computer results are shown in Figure 8. The vacuum thrust coefficient is about one percent higher than the design value, but the design method was assumed as a simple wave throughout the whole flow field. An internal-external expansion plug nozzle was also computed. The result and the flow pattern are shown in Figure 9.

When a nozzle contour is not well described or a compression region occurs in the flow field, the characteristics overlap indicating that a shock is being formed in that region. If the shock is weak, the present program carries on the calculations by assuming an isentropic process. A shock routine must be developed to analyze a nozzle with a strong shock. The Rankine-Hugoniot equations are normally used for this purpose.

In the derivation of the transonic solution, the second degree of the velocity components was ignored. Therefore a significant amount of error would be introduced to the result if the Mach number of the starting line were high. In the case shown in Figure 9, two percent of error in vacuum thrust was found when the initial Mach number changed from 1.05 to 1.15.

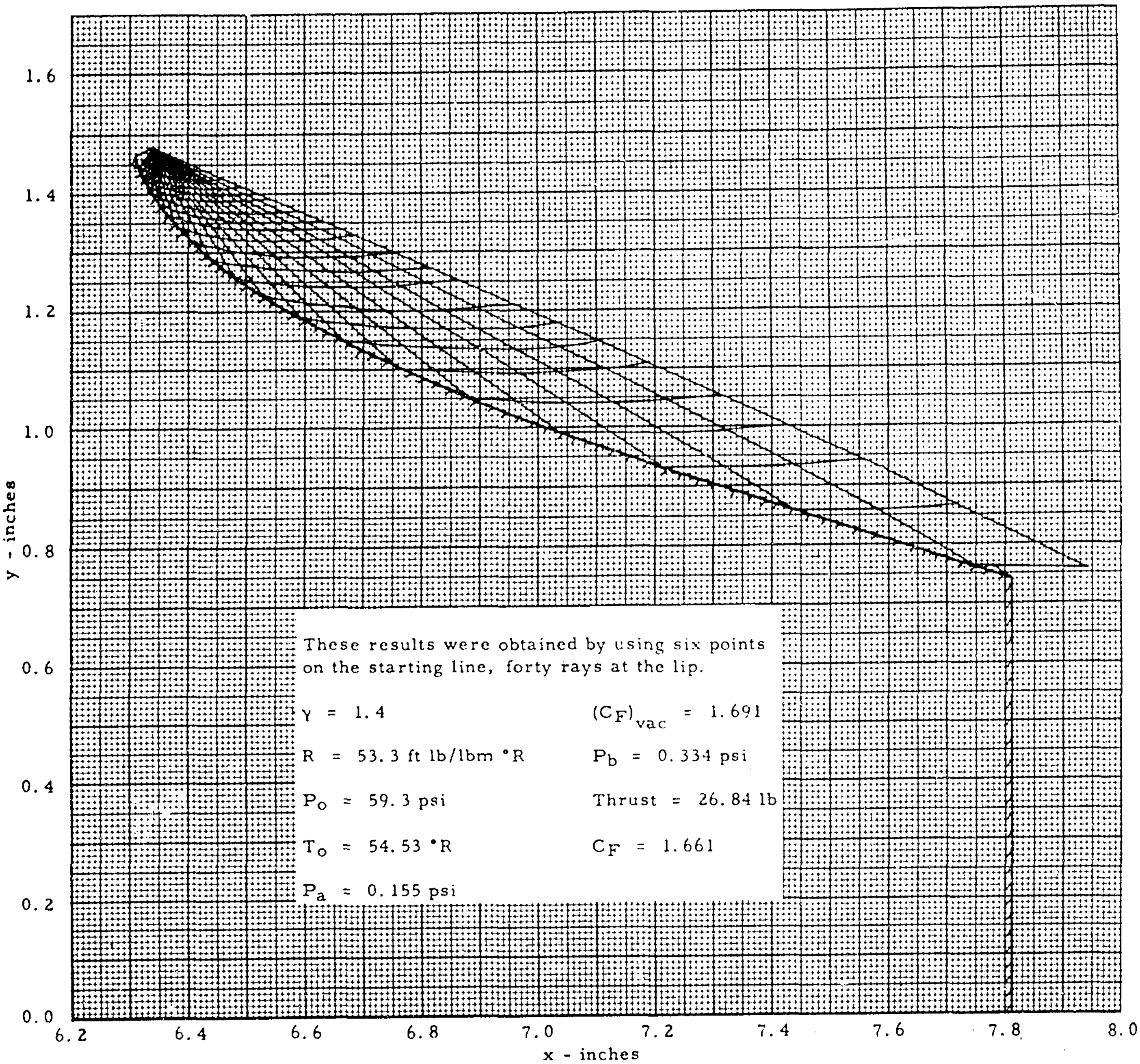

Figure 8. Flow Field of an External Expansion Plug Nozzle

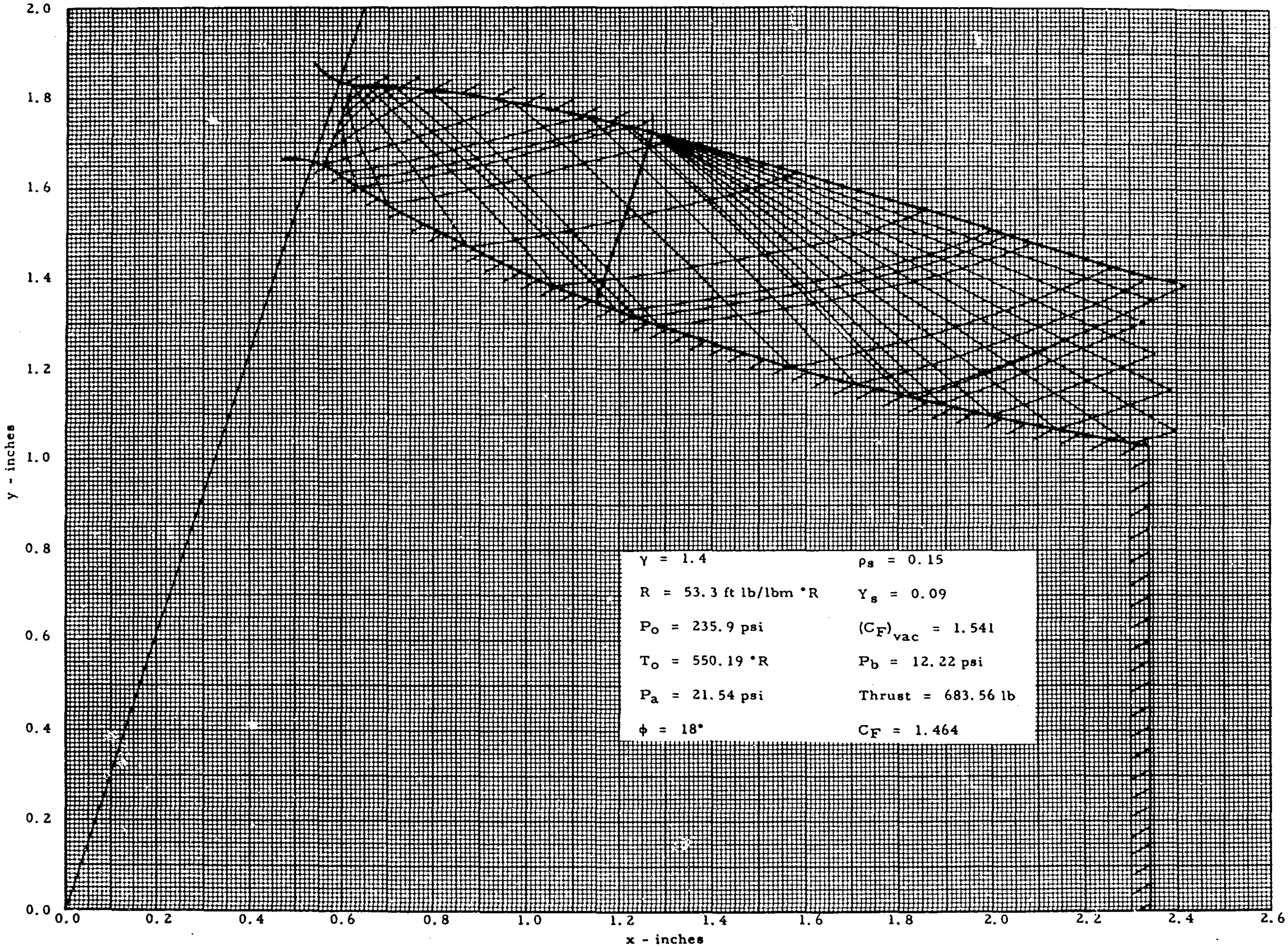

Figure 9. Flow Field of an Internal-External Expansion Plug Nozzle

This program is suitable for a basic study of plug nozzle performance. In order to improve the quality of the result, the following items are recommended for future work.

1. To develop a shock routine there will be no difficulty because rotational flow was assumed in the present program.
2. To include real gas equations in the computation.
3. To take into account the friction loss on the nozzle walls.

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3. R. Sauer, "General Characteristics of the Flow Through Nozzle at Near Critical Speed", NACA Technical Memorandum No. 1147, June 1947.

4. H. H. Korst, R. H. Page, M. E. Childs, "A Theory for Base Pressures in Transonic and Supersonic Flow", ME Technical Note 392-2, Engineering Experiment Station, University of Illinois, March 1955.

5. H. M. Darwell, H. Badham, "Shock Formation in Conical Nozzle", AIAA Journal, Vol. 1, Number 8, August 1963.

6. C. C. Lee, "FORTRAN Program for Plug Nozzle Design", Brown Engineering Company, Technical Note R-41, March 1963.

APPENDIX

DESCRIPTION OF DATA INPUT AND OUTPUT

Input

This program requires the following input data:

(1) Nozzle components

FE (ϕ^*)	--	throat plane inclined angle; degrees for internal-external expansion
	--	radians for external expansion
ROS (ρ_s)	--	used only for internal-external expansion
	--	equals 0. for external expansion
YS (y_s)	--	radius of nozzle throat; used only for internal-external expansion
	--	equals 0. for external expansion
GAM (γ)	--	ratio of specific heats
XM (M_{est})	--	initial Mach number
P (P_0)	--	total pressure
T (T_0)	--	total temperature
RT	--	radius from the throat to origin, used only for internal-external expansion
	--	equals 0. for external expansion
R	--	gas constant
N	--	number of points on starting line, must be <100.
N1	--	number of lower wall contour points, must be <100.
N2	--	number of upper wall contour points, must be <100.

(2) A title or job-description card

(3) N1 lower wall contour points given as Cartesian coordinates

(4) N2 upper wall contour points given as Cartesian coordinates

(5) KK -- If input is in feet kk = 0
-- If input is in inches kk = 1

(6) KODE

(a) If KODE = 1, read starting line for an internal-external plug nozzle expansion

(b) If KODE = 2, compute starting line for an internal-external plug nozzle expansion

(c) If KODE = 3, compute starting line for an external plug nozzle expansion

(7) KODE used for external expansion only

(a) If KODE = 1 use standard starting line calculations

(b) If KODE = 2 use a special option in calculating the starting line: $M_{est} = M_E$ and $\phi = \theta_E$.

(8) NU (η) -- the number of corner rays to be computed, must be < 100.
PA (P_a) -- Ambient Pressure
KKD -- If KKD = 0, PA is in lbs/sq ft
-- If KKD = 1, PA is in lbs/sq in

Input cases can be stacked and processed several at a time. If a bad data case is found, the remaining data cases will not be processed. This is due to the computer system, not the program.

INPUT DATA AND FORMAT

Format	Data	Description	No. of Cards
(5E15.8)	FE, ROS, YS, GAM, XM	Symbols are defined in (1) of Input description	1
(4E15.8, 3I2)	P, T, RT, R, N, N1, N2	Symbols are defined in (1) of Input description	1
(13A6)	Title or Job Description Card	Maximum of 78 alphanumeric characters (col. 1-78)	1
(2E15.8)	X, Y	Contour Cartesian coordinates lower wall points followed by upper wall points	1+: one for each contour point = N1 + N2
(I2)	KK	Defined in (5) of Input description	1
(I2)	KODE	Defined in (6) of Input description	1
(I2)	KODE	Omit this card for Internal-External expansion defined in (7) of Input description	1
(I2. E15.8, I2)	NU, PA, KKD	Symbols are defined in (8) of Input description	1

INPUT CARD FORMAT

2	15	17	19	30	45	60	62	64	66	75
FE		ROS		YS	GAM	XM				
P		T		RT	R	N	N1	N2		

TITLE CARD

X	Y	(N1 + N2 contour point cards)
↓	↓	

KK	
KODE	
KODE	(omit for internal-external expansion)

	2	17	19
NU	PA	KKD	

SAMPLE DATA
INTERNAL-EXTERNAL EXPANSION

```
2        15 17 19     30           45           60 62 64 66    75
+0.18000000E+02+0.15000000E+02+0.90000000E-01+0.14000000E+01+0.10500000E+01

+0.23590000E+03+0.55019000E+03+0.18400000E+01+0.53300000E+02069129

      16.03 PER CENT PLUG NOZZLE

+0.54200000E+00+0.16620000E+01   (120 = 91 + 29 cards with this format describing

                                  lower and upper wall contour)

01

02

10+0.21540000E+0201
```

SAMPLE DATA
EXTERNAL EXPANSION

```
2      15 17 19      30      45      60 62 69 66      75
-0.13424000E+01+0.00000000E+00-0.00000000E+00+0.14000000E+01+0.12000000E+01

+0.59300000E+02+0.54530000E+03+0.00000000E+00+0.53300000E+02041601

     EXTERNAL EXPANSION PLUG NOZZLE

+0.63112049E+0  +0.14663178E+01  (17 = 16 + 1 cards with this format describing
    ↓               ↓
                                 lower wall and upper corner contour points)

01

03

01

30+0.15500000E+0001
```

Output

(1) Units of variables

(2) Job title

(3) Input conditions

(4) Upper wall contour

(5) Lower wall contour

(6) Starting line points

X	Y	M	THETA	T	P
↓	↓	↓	↓	↓	↓

where X, Y are Cartesian coordinates; M is Mach number, THETA is flow angle, T is temperature (°R), and P is pressure

(7) Internal expansion

(a) Field routine points

X	Y	M	THETA	T	P	ITR
↓	↓	↓	↓	↓	↓	↓

where ITR is the number of iterations before convergence in calculations

(b) Body point routine point

X	Y	M	THETA	T	P	ITR

Field and body points alternate until the last point on the upper wall contour is reached

(8) External expansion

(a) Insert point

X	Y	M	THETA	T	P

(b) Corner point

X	Y	M	THETA	T	P

(c) Right running characteristics

X	Y	M	THETA	T	P
↓	↓	↓	↓	↓	↓

(d) Field routine points

X	Y	M	THETA	T	P	ITR
↓	↓	↓	↓	↓	↓	↓

(e) Body point routine point

X Y M THETA T P ITR

Field and body points alternate until the last point on the lower wall contour is reached or until the network is completed.

(f) Insert point

X Y M THETA T P

(g) Corner point

X Y M THETA T P

(9) Thrust distribution along the plug

(a) SUMM

(b) CFI

(c) Mass flow rate

(d)

X	Y	T	CF
↓	↓	↓	↓

for each point on the lower wall

(e) AT -- throat area

(f) PB -- base pressure

(g) TVAC -- vacuum thrust

(h) CFVAC -- Vacuum thrust coefficient

(i) THRUST -- real thrust

(j) CF REAL -- real thrust coefficient

(k) End of job

LIST OF FORTRAN PROGRAM

PLUG NOZZLE ANALYSIS BY USING THE METHOD OF CHARACTERISTICS

MAIN PROGRAM

```
      DIMENSION YP(400),XP(400),TH(400),XMP(400),TP(400),PP(400),
     1RXM(200),RTH(200),RTP(200),RPP(200),VLP(400)
      DIMENSION XB1(100),XB2(100),YB1(100),YB2(100)
      DIMENSION FRX(50),FRY(50),FRV(50),FRT(50),FRP(50),FRTH(50),
     1FUX(100),FUY(100),FUP(100),FLX(200),FLY(200),FLP(200)
      DIMENSION ZC(100),ZJ(100),XM1(100),PBP1(100)
      COMMON YP,XP,TH,XMP,TP,PP,RXM,RTH,RTP,RPP,VLP,ROS,YS,GAM,GM1,G,
     1XM,N,P,T,R,L,M,J,N2,XXB2,YYB2,NU,KNT,GP1,FE,RT
     1,PA
      COMMONZC,ZJ,XM1,PBP1,NQ
      READ(5,52)NQ
      READ(5,1003)(ZJ(I),ZC(I),I=1,NQ)
   78 READ(5,1001)FE,ROS,YS,GAM,XM,P,T,RT,R,N,N1,N2
  300 FORMAT(1H1,54X,22HPLUG   NOZZLE   ANALYSIS/1H0,44X,43HBY   USING   THE
     1   METHOD   OF   CHARACTERISTICS////1H0,10X,5HUNITS///1H0,10X,16HCOOR
     1DINATES   X,Y,14X,4HIN. /1H0,10X,25HINCLINED THROAT ANGLE(FE),5X,7H
     1DEGREES/1H0,10X,8HPRESSURE,22X,9HLBF/IN*IN/1H0,10X,11HTEMPERATURE,
     119
     1X,15HDEGREES RANKINE/1H0,10X,16HGAS CONSTANT (R),14X,26HFT LBF/LBM
```

LIST OF FORTRAN PROGRAM

PLUG NOZZLE ANALYSIS BY USING THE METHOD OF CHARACTERISTICS

```
      1 DEGREES RANKINE/1H0,10X,4HAREA,26X,5HIN*IN/1H0,10X,6HTHRUST,24X,3
      1HLBF)
  301 FORMAT(13A6)
  302 FORMAT(1H0,13A6)
  303 FORMAT(1H0,10X,17HINPUT  CONDITIONS///1H0,10X,3HFE=E15.8/1H0,10X,3
      1HRT=E15.8/1H0,10X,3HYS=E15.8/1H0,10X,5HRHOS=E15.8/1H0,10X,6HGAMMA=
      1E15.8/1H0,10X,5HMEST=E15.8/1H0,10X,2HR=E15.8/1H0,10X,3HPO=E15.8/1H
      10,10X,3HTO=E15.8)
       READ(5,301)A1,A2,A3,A4,A5,A6,A7,A8,A9,A10,A11,A12,A13
       WRITE(6,300)
       WRITE(6,302)A1,A2,A3,A4,A5,A6,A7,A8,A9,A10,A11,A12,A13
       READ(5,1003)(XB1(I),YB1(I),I=1,N1),(XB2(I),YB2(I),I=1,N2)
       WRITE(6,303)FE,RT,YS,ROS,GAM,XM,R,P,T
       WRITE(6,1006)
       WRITE(6,1007)(XB2(I),YB2(I),I=1,N2)
       WRITE(6,1008)
       WRITE(6,1007)(XB1(I),YB1(I),I=1,N1)
       READ(5,52)KK
       IF(KK.EQ.0)GO TO 401
       P=P*144.
```

LIST OF FORTRAN PROGRAM

PLUG NOZZLE ANALYSIS BY USING THE METHOD OF CHARACTERISTICS

```
      YS=YS/12.
      ROS=ROS/12.
      RT=RT/12.
      DO 402 K=1,N1
      XB1(K)=XB1(K)/12.
  402 YB1(K)=YB1(K)/12.
      DO 403 K=1,N2
      XB2(K)=XB2(K)/12.
  403 YB2(K)=YB2(K)/12.
  401 XXB2=XB2(N2)
      YYB2=YB2(N2)
 1006 FORMAT(1H0,18HUPPER WALL CONTOUR/1H0,7X,1HX,16X,1HY)
 1007 FORMAT(1H0,2(E15.8,2X))
 1008 FORMAT(1H0,18HLOWER WALL CONTOUR/1H0,7X,1HX,16X,1HY)
      NF=2
      NU=0
      KNT=1
      GM1=GAM-1.
      GP1=GAM+1.
      G=32.2
```

LIST OF FORTRAN PROGRAM

PLUG NOZZLE ANALYSIS BY USING THE METHOD OF CHARACTERISTICS

```
      WRITE(6,1002)
      READ(5,52)KODE
C     KODE=1--READ START LINE
C     KODE=2--COMPUTE START LINE
   52 FORMAT(I2)
      GO TO (53,54,84,500),KODE
  500 KODE=3
      GO TO 53
   84 CALL STL2(XB1,YB1,N1)
      AT=3.1415927*(YB2(1)+YB1(1))*SQRT((XB2(1)-XB1(1))**2+(YB2(1)-YB1(1
     1))**2)
      GO TO 2
   53 READ(5,1005)(XP(J),YP(J),TH(J),XMP(J),TP(J),PP(J),J=1,N)
      READ(5,1005)AT
 1005 FORMAT(6E13.6)
      DO 99 J=1,N
      WRITE(6,79)XP(J),YP(J),XMP(J),TH(J),TP(J),PP(J)
   79 FORMAT(1H0,6(3X,E15.8))
   99 TH(J)=TH(J)*.01745329
      GO TO 2
```

LIST OF FORTRAN PROGRAM

PLUG NOZZLE ANALYSIS BY USING THE METHOD OF CHARACTERISTICS

```
54 FE=FE*.01745329
   RG1=(RT+YS)*COS(FE)
   RG2=(RT-YS)*COS(FE)
   X1=(RT+YS)*SIN(FE)
   X2=(RT-YS)*SIN(FE)
   AT=3.1415927*(RG1+RG2)*SQRT((X1-X2)**2+(RG1-RG2)**2)
   CALL SLRTN
 2 CP=GAM*R/GM1
   DO 60 J=1,N
   H=CP*TP(J)*G
   A=SQRT(GM1*H)
60 VLP(J)=XMP(J)*A
   K=N
   DO 61 J=1,N
   FRX(J)=XP(K)
   FRY(J)=YP(K)
   FRV(J)=VLP(K)
   FRT(J)=TP(K)
   FRP(J)=PP(K)
   FRTH(J)=TH(K)
```

LIST OF FORTRAN PROGRAM

PLUG NOZZLE ANALYSIS BY USING THE METHOD OF CHARACTERISTICS

```
 61 K=K-1
    FLX(1)=XP(1)
    FLY(1)=YP(1)
    FLP(1)=PP(1)
    FUX(1)=XP(N)
    FUY(1)=YP(N)
    FUP(1)=PP(N)
    GO TO (222,222,74),KODE
222 M=N+N-1
    L=N+1
    J=0
    CALL FLDRTN (1)
    M=1
    L=N+1
    CALL BPRTN(1,XB1,YB1,N1)
    FLX(NF)=XP(1)
    FLY(NF)=YP(1)
    FLP(NF)=PP(1)
    M=N-1
    L=2
```

LIST OF FORTRAN PROGRAM

PLUG NOZZLE ANALYSIS BY USING THE METHOD OF CHARACTERISTICS

```
      J=N
      CALL FLDRTN (1)
      M=N
      L=N+N-1
      CALL BPRTN(2,XB2,YB2,N2)
      GO TO 75
   74 CALL BPRTN(3,XB2,YB2,N2)
      NR=N+1
   86 M=NR-1
      L=2
      J=NR-1
      CALL FLDRTN(3)
      M=1
      L=2
      CALL BPRTN(1,XB1,YB1,N1)
      FLX(NF)=XP(1)
      FLY(NF)=YP(1)
      FLP(NF)=PP(1)
      NF=NF+1
      NR=NR+1
```

LIST OF FORTRAN PROGRAM

PLUG NOZZLE ANALYSIS BY USING THE METHOD OF CHARACTERISTICS

```
      IF(NR-(N+NU))88,88,6
   88 IF(J-666)86,6,86
   75 FUX(NF)=XP(N)
      FUY(NF)=YP(N)
      FUP(NF)=PP(N)
      NF=NF+1
      IF(NU)222,222,20
 1001 FORMAT(5E15.8/4E15.8,3I2)
 1003 FORMAT(2E15.8)
 1002 FORMAT(1H0,///,1H0,10X,1HX,17X,1HY,17X,1HM,13X,5HTHETA,13X,
     11HT,17X,1HP,10X,3HIIR)
   20 KZ=1
      NG=NF-1
      NBP=N-2+NU
      NB2=NBP/2
      IF(( N/2)*2-N)14,13,13
   14 IF(2*NB2-NBP)22,1,1
   13 IF(2*NB2-NBP)1,22,22
    1 NZ=(NBP+N-1)/2
      NY=1
```

LIST OF FORTRAN PROGRAM

PLUG NOZZLE ANALYSIS BY USING THE METHOD OF CHARACTERISTICS

```
      GO TO 3
   22 NZ=(NBP+N)/2
      NY=2
    3 M=N+N-1
      JL=N
      NB=1
      L=N+1
      J=0
      CALL FLDKTN(2)
      IJ=1
      I=0
      M=1
      L=N+IJ
      CALL BPRTN(1,XB1,YB1,N1)
    4 FLX(NF)=XP(1)
      FLY(NF)=YP(1)
      FLP(NF)=PP(1)
      NF=NF+1
   44 IF(J-666)45,6,45
   45 J=L-1
```

LIST OF FORTRAN PROGRAM

PLUG NOZZLE ANALYSIS BY USING THE METHOD OF CHARACTERISTICS

```
      L=2
      M=N+I
      IF(M-L)6,5,5
    5 GO TO (9,12),KZ
    9 IF(M-NZ)7,8,7
    8 KZ=2
      GO TO (11,12),NY
   11 CALL FLDRTN(2)
      J=0
      L=N+IJ
      M=2*N+2*I
      CALL FLDRTN(2)
      NY=2
      M=1
      L=N+IJ
      CALL BPRTN(1,XB1,YB1,N1)
      GO TO 4
   12 CALL FLDRTN(2)
      J=0
      L=N+IJ
```

PLUG NOZZLE ANALYSIS BY USING THE METHOD OF CHARACTERISTICS

```
      M=2*N+2*I-1
      CALL FLDRTN(2)
      M=1
      L=N+IJ
      CALL BPRTN(1,XB1,YB1,N1)
      IJ=IJ-1
      I=I-1
      GO TO 4
    7 CALL FLDRTN(2)
      J=0
      L=N+IJ
      M=2*N+2*I
      CALL FLDRTN(2)
      M=1
      L=N+IJ
      CALL BPRTN(1,XB1,YB1,N1)
      IJ=IJ+1
      I=I+1
      GO TO 4
    6 SUMP=0.
```

LIST OF FORTRAN PROGRAM

PLUG NOZZLE ANALYSIS BY USING THE METHOD OF CHARACTERISTICS

```
      WRITE(6,305)
  305 FORMAT(1H0/1H0,34HTHRUST DISTRIBUTION ALONG THE PLUG)
      IF(KODE .EQ. 3) GO TO 207
      I=NG
      NG=NG-1
      DO 64 K=1,NG
      I=I-1
      P12=(FUP(I)+FUP(I+1))/2.
      A12=3.1415927*(FUY(I)+FUY(I+1))*SQRT((FUX(I)-FUX(I+1))**2+(FUY(I)
     1-FUY(I+1))**2)
      FE12=ATAN((FUX(I)-FUX(I+1))/(FUY(I+1)-FUY(I)))
      PI=P12*A12*COS(FE12)
      IF(FUY(I+1)-FUY(I))71,76,70
   76 PI=0.
      GO TO 70
   71 PI=-PI
   70 SUMP=SUMP+PI
   64 CONTINUE
  207 SUMM=0.
      SUMV=0.
```

LIST OF FORTRAN PROGRAM

PLUG NOZZLE ANALYSIS BY USING THE METHOD OF CHARACTERISTICS

```
      NZ=N-1
      DO 91 I=1,NZ
      P12=(FRP(I)+FRP(I+1))/2.
      T12=(FRT(I)+FRT(I+1))/2.
      RO12=P12/(R*T12)
      A12=3.1415927*(FRY(I)+FRY(I+1))*SQRT((FRX(I)-FRX(I+1))**2+(FRY(I)
     1-FRY(I+1))**2)
      TH12=(FRTH(I)+FRTH(I+1))/2.
      FE12=ATAN((FRX(I)-FRX(I+1))/(FRY(I+1)-FRY(I)))
      V12=(FRV(I)+FRV(I+1))/2.
      SQ=FE12-TH12
      VM=RO12*V12*A12*COS(SQ)
      VMOM=VM/G*V12*COS(TH12)
      VMOMP=P12*A12*COS(FE12)
      VMO=VMOM+VMOMP
      SUMM=SUMM+VMO
      SUMV=SUMV+VM
   91 CONTINUE
      SUMM=SUMP+SUMM
      CFI=SUMM/(P*AT)
```

LIST OF FORTRAN PROGRAM

PLUG NOZZLE ANALYSIS BY USING THE METHOD OF CHARACTERISTICS

```
      WRITE(6,96)SUMM,CFI
      WRITE(6,400)SUMV
  400 FORMAT(1H0,15HMASS FLOW RATE=E15.8)
   96 FORMAT(1H0,5HSUMM=E15.8,3X,4HCFI=E15.8)
      SUMP=0.
      NB=NF-2
      DO 92 I=1,NB
      IF(SUMP)602,602,601
  602 CONTINUE
      IF(FLX(1)-FLX(I+1))600,92,92
  600 IF(I.EQ.1)GO TO 601
      P12=(FLP(1)+FLP(I+1))/2.
      A12=3.1415927*(FLY(1)+FLY(I+1))*SQRT((FLX(1)-FLX(I+1))**2
     1+(FLY(1)-FLY(I+1))**2)
      FE12=ATAN((FLX(1)-FLX(I+1))/(FLY(I+1)-FLY(1)))
      PI=P12*A12*COS(FE12)
      IF(FLY(1)-FLY(I+1))72,73,73
  601 P12=(FLP(I)+FLP(I+1))/2.
      A12=3.1415927*(FLY(I)+FLY(I+1))*SQRT((FLX(I)-FLX(I+1))**2
     1+(FLY(I)-FLY(I+1))**2)
```

LIST OF FORTRAN PROGRAM

PLUG NOZZLE ANALYSIS BY USING THE METHOD OF CHARACTERISTICS

```
      FE12=ATAN((FLX(I)-FLX(I+1))/(FLY(I+1)-FLY(I)))
      PI=P12*A12*COS(FE12)
      IF(FLY(I)-FLY(I+1))72,73,73
   72 PI=-PI
   73 SUMP=SUMP+PI
      TOT=SUMM+SUMP
      CF=TOT/(P*AT)
      WRITE(6,94)FLX(I+1),FLY(I+1),TOT,CF
   92 CONTINUE
   94 FORMAT(1H0,2HX=E15.8,3X,2HY=E15.8,3X,2HT=E15.8,3X,3HCF=E15.8)
      AX=AT*144.
      WRITE(6,93)AX
   93 FORMAT(1H0,3HAT=E15.8)
      DUMMY=XMP(1)
      CALL CONVR (1,DUMMY,W1)
      THA=TH(1)+W1
      IF(THA)201,202,202
  201 WRITE(6,200)
      PB=PA
      GO TO 203
```

LIST OF FORTRAN PROGRAM

PLUG NOZZLE ANALYSIS BY USING THE METHOD OF CHARACTERISTICS

```
200 FORMAT(1H0,10X,34HFLOW BREAK-AWAY FROM UPSTREAM WALL/1H0,10X,9HSET
   1 PB=PA)
202 CALL CONVR(2,XMRED,THA)
    PRED=1./((1.+GM1/2.*XMRED**2)**(GAM/GM1))
    PDA=(1.+GM1/2.*XMP(1)**2)**(GAM/GM1)
    CALL BPRS
    PB=TABLE1(PBP1,XM1,XMRED,NC)
    P2PR=PB
    P2P1=P2PR*PDA*PRED
    IF(P2P1-1.)204,205,205
205 WRITE(6,206)
    PB=PA
    GO TO 203
206 FORMAT(1H0,10X,22HTHEORY BECOMES INVALID/1H0,10X,9HSET PB=PA)
204 PB=PP(1)*P2P1
203 CONTINUE
    TVAC=TOT+PB*3.1415927*FLY(NF-1)**2
    CFVAC=TVAC/(P*AT)
    AE=3.1415927*YB2(N2)**2
    THRUST=TVAC-PA*AE
```

LIST OF FORTRAN PROGRAM

PLUG NOZZLE ANALYSIS BY USING THE METHOD OF CHARACTERISTICS

```
      CFREAL=THRUST/(P*AT)
      PB=PB/144.
      WRITE(6,6111)PB,TVAC,CFVAC,THRUST,CFREAL
 6111 FORMAT(1H0,3HPB=E15.8/1H0,5HTVAC=E15.8/1H0,6HCFVAC=E15.8/1H0,7HTHR
     1UST=E15.8/1H0,7HCFREAL=E15.8)
   51 WRITE(6,2000)
 2000 FORMAT(1H0,///,20X,10HEND OF JOB)
      GO TO 78
      END
      SUBROUTINE SLRTN
      DIMENSION YP(400),XP(400),TH(400),XMP(400),TP(400),PP(400),
     1RXM(200),RTH(200),RTP(200),RPP(200),VLP(400)
      COMMON YP,XP,TH,XMP,TP,PP,RXM,RTH,RTP,RPP,VLP,ROS,YS,GAM,GM1,G,
     1XM,N,P,T,R,L,M,J,N2,XXB2,YYB2,NU,KNT,GP1,FE,RT
     1,PA
      IF(N-10)50,51,51
   50 XN=N
      GO TO 52
   51 XN=N-4
   52 CONTINUE
```

LIST OF FORTRAN PROGRAM

PLUG NOZZLE ANALYSIS BY USING THE METHOD OF CHARACTERISTICS

```
      PST=P*(2./GP1)**(GAM/GM1)
      TST=T*(2./GP1)
      A=SQRT(1./(GP1*ROS*YS))
      EPP=YS/6.*SQRT(GP1*YS/ROS)
      FEO=ATAN(EPP/RT)
      FI=FE+FEO
      RTO=SQRT(EPP*EPP+RT*RT)
      HH=RTO*SIN(FI)
      HK=RTO*COS(FI)
      XMES=SQRT((GP1/2.*XM*XM)/(1.+GM1/2.*XM*XM))
      XPP=(XMES-1.)/A
      PHA=ARSIN((EPP+XPP)/ROS)
      YL=YS+(ROS-ROS*COS(PHA))
      DYL=2.*YL/(XN-1.)/2.
      DO 1 I=1,N
      IF(N-10)44,9,9
    9 IF(I-6)4,2,3
    3 IF(I-(N-3))6,5,7
    2 DYL=2.*DYL
      GO TO 6
```

LIST OF FORTRAN PROGRAM

PLUG NOZZLE ANALYSIS BY USING THE METHOD OF CHARACTERISTICS

```
 5 DYL=DYL/2.
   GO TO 7
 6 YPP=FLOAT(I-3)*DYL-YL
   GO TO 8
 7 YPP=FLOAT(I+N-10)*DYL-YL
   GO TO 8
44 YPP=FLOAT(I-1)*DYL*2.-YL
   GO TO 8
 4 YPP=FLOAT(I-1)*DYL-YL
 8 U=A*XPP+GP1/2.*A*A*YPP*YPP
   V=A*A*GP1*(XPP*YPP+GP1/6.*A*YPP**3)
   XMS=SQRT((1.+U)**2+V*V)
   THX=ATAN(V/(1.+U))
   XMP(I)=SQRT(2./GP1*XMS*XMS/(1.-GM1/GP1*XMS*XMS))
   PP(I)=PST/((2./GP1)**(GAM/GM1)*(1.+GM1/2.*XMP(I)**2)
  1**(GAM/GM1))
   TP(I)=TST/((1.+GM1/2.*XMP(I)**2)*(2./GP1))
   XP(I)=XPP*COS(-FI)-YPP*SIN(-FI)+HH
   YP(I)=YPP*COS(-FI)+XPP*SIN(-FI)+HK
   TH(I)=THX-FI
```

LIST OF FORTRAN PROGRAM

PLUG NOZZLE ANALYSIS BY USING THE METHOD OF CHARACTERISTICS

```
      THH=TH(I)*57.29578
      QX=XP(I)*12.
      QY=YP(I)*12.
      QP=PP(I)/144.
    1 WRITE(6,102)QX,QY,XMP(I),THH,TP(I),QP
  102 FORMAT(1H0,6(3X,E15.8))
      RETURN
      END
       SUBROUTINE STL2(XB,YB,N1)
      DIMENSION YP(400),XP(400),TH(400),XMP(400),TP(400),PP(400),
     1RXM(200),RTH(200),RTP(200),RPP(200),VLP(400)
      COMMON YP,XP,TH,XMP,TP,PP,RXM,RTH,RTP,RPP,VLP,ROS,YS,GAM,GM1,G,
     1XM,N,P,T,R,L,M,J,N2,XXB2,YYB2,NU,KNT,GP1,FE,RT
     1,PA
      DIMENSION XB(100),YB(100)
      PI=3.1415927
      READ(5,11)KODE
      GO TO (12,13),KODE
   11 FORMAT(I2)
   13 TH(1)=FE
```

LIST OF FORTRAN PROGRAM

PLUG NOZZLE ANALYSIS BY USING THE METHOD OF CHARACTERISTICS

```
      FE=FE-ARSIN(1./XM)
      GO TO 16
   12 THST=FE+PI/2.
      FE=FE+(PI/2.-ARSIN(1./XM)+SQRT(GP1/GM1)*ATAN(SQRT(GM1/GP1*(XM*XM-1
     1.)))-ATAN(SQRT(XM*XM-1.)))
   16 JJ=1
      TSC=SIN(PI+FE)/COS(PI+FE)
    1 YD=YB(JJ+1)-YB(JJ)
      XD=XB(JJ+1)-XB(JJ)
      XA=1./(YD/XD-    TSC)*(YYB2-YB(JJ)+YD/XD*XB(JJ)-XXB2*TSC)
      IF(XA-XB(JJ))3,2,2
    2 IF(XA-XB(JJ+1))4,4,5
    5 JJ=JJ+1
      GO TO 1
    3 WRITE(6,6)XA,XB(JJ),XB(JJ+1)
    6 FORMAT(1H0,3E15.8)
      STOP
    4 YP(1)=YB(JJ)+(XA-XB(JJ))/XD*YD
      XP(1)=XA
      GO TO (14,15),KODE
```

PLUG NOZZLE ANALYSIS BY USING THE METHOD OF CHARACTERISTICS

```
14 TH(1)=THST+SQRT(GP1/GM1)*ATAN(SQRT(GM1/GP1*(XM**2-1.)))-ATAN(SQRT(
  1XM**2-1.))
15 XMP(1)=XM
   PP(1)=P/((1.+GM1/2.*XM**2)**(GAM/GM1))
   TP(1)=T/(1.+GM1/2.*XM**2)
   THP=TH(1)*57.29578
   QX=XP(1)*12.
   QY=YP(1)*12.
   QP=PP(1)/144.
   WRITE(6,10)QX,QY,XMP(1),THP,TP(1),QP
   XN=N
   XN=XN-1.
   DX=(XXB2-XA)/XN
   DO 9 MM=2,N
   XP(MM)=XP(MM-1)+DX
   YP(MM)=YP(MM-1)+(YYB2-YP(1))/(XXB2-XP(1))*DX
   XMP(MM)=XM
   TH(MM)=TH(1)
   TP(MM)=TP(1)
   PP(MM)=PP(1)
```

PLUG NOZZLE ANALYSIS BY USING THE METHOD OF CHARACTERISTICS

```
      QX=XP(MM)*12.
      QY=YP(MM)*12.
    9 WRITE(6,10)QX,QY,XM,THP,TP(1),QP
   10 FORMAT(1H0,6(3X,E15.8))
      RETURN
      END
      SUBROUTINE BPRTN(KODE,X,Y,NX)
      DIMENSION YP(400),XP(400),TH(400),XMP(400),TP(400),PP(400),
     1RXM(200),RTH(200),RTP(200),RPP(200),VLP(400)
      DIMENSION X(100),Y(100)
      COMMON YP,XP,TH,XMP,TP,PP,RXM,RTH,RTP,RPP,VLP,ROS,YS,GAM,GM1,G,
     1XM,N,P,T,R,L,M,J,N2,XXB2,YYB2,NU,KNT,GP1,FE,RT
     1,PA
      IF(KODE .EQ. 3) GO TO 65
      ITR=1
      WRITE(6,1004)
 1004 FORMAT(1H0,40X,18HBODY POINT ROUTINE)
      CP=GAM*R/GM1
      H1=CP*TP(L)*G
      A1=SQRT(GM1*H1)
```

LIST OF FORTRAN PROGRAM

PLUG NOZZLE ANALYSIS BY USING THE METHOD OF CHARACTERISTICS

```
      V1=XMP(L)*A1
      B1=ARSIN(A1/V1)
      H=H1+V1*V1/2.
      PS=P*(2./GP1)**(GAM/GM1)
      TS=T*(2./GP1)
      S1=CP*ALOG((TP(L)/TS)/((PP(L)/PS )**(GM1/GAM)))
      SB=CP*ALOG((TP(M)/TS)/((PP(M)/PS )**(GM1/GAM)))
      DO 1 I=1,NX
      K=I
      IF(X(I)-XP(L))1,1,2
    1 CONTINUE
   20 GO TO (60,61,52),KODE
   60 KEY=1
      GO TO 23
   61 KEY=2
      GO TO 23
   23 K=NX
      TH3=ATAN((Y(K)-Y(K-1))/(X(K)-X(K-1)))
      B3=ARSIN(1./XMP(M))
      A3=SQRT(GM1*CP*TP(M)*G)
```

```
      V3=XMP(M)*A3
      T3=TP(M)
      TH4=TH(M)
   58 CONTINUE
   37 GO TO (34,35),KEY
   35 C1=(TH3+B3+TH4+B4)/2.
      GO TO 43
   34 C1=(TH3-B3+TH4-B4)/2.
   43 C1=SIN(C1)/COS(C1)
      B=((Y(K)-YP(L))-(X(K)-XP(L))*C1)/((YP(M)-YP(L))-(XP(M)-XP(L))*C1)
      X4=XP(L)+B*(XP(M)-XP(L))
      Y4=YP(L)+B*(YP(M)-YP(L))
      XM4=XMP(L)+B*(XMP(M)-XMP(L))
      TH4=TH(L)+B*(TH(M)-TH(L))
      T4=TP(L)+B*(TP(M)-TP(L))
      P4=PP(L)+B*(PP(M)-PP(L))
      S4=S1+B*(SB-S1)
      TH44=TH4*57.29578
      A4=SQRT(GM1*CP*T4*G)
      V4=XM4*A4
```

PLUG NOZZLE ANALYSIS BY USING THE METHOD OF CHARACTERISTICS

```
      B4=ARSIN(1./XM4)
      GO TO (45,46),KEY
   46 C3=COS((TH3+B3+TH4+B4)/2.)
      GO TO 47
   45 C3=COS((TH3-B3+TH4-B4)/2.)
   47 C1=(B3+B4)/2.
      C2=COS(C1)
      C1=SIN(C1)
      GO TO (48,49),KEY
   49 V3=V4+((V3+V4)/2.*C1/C2)*((TH3-TH4+C1*SIN((TH3+TH4)/2.)/(((Y(K)
     1+Y4)/2.*C3)*(X(K)-X4)-((T3+T4)/2.)/((((A3+A4)/2.)**2)*C1*C2 *
     1(SB-S4)*G))
      GO TO 50
   48 V3=V4+((V3+V4)/2.*C1/C2)*((TH4-TH3+C1*SIN((TH3+TH4)/2.)/(((Y(K)
     1+Y4)/2.*C3)*(X(K)-X4)-((T3+T4)/2.)/((((A3+A4)/2.)**2)*C1*C2 *
     1(SB-S4)*G))
   50 H3=H-.5*V3*V3
      A3=SQRT(GM1*H3)
      B3=ARSIN(A3/V3)
      XMP3=V3/A3
```

LIST OF FORTRAN PROGRAM

PLUG NOZZLE ANALYSIS BY USING THE METHOD OF CHARACTERISTICS

```
      T3=H3/(CP*G)
      P3=PP(M)*(T3/TP(M))**(GAM/GM1)
      THP=TH3*57.29578
      IF(ABS(B-BP)-.000001)56,56,57
   57 BP=B
      GO TO 58
   56 WRITE(6,205)
  205 FORMAT(1H0,12HINSERT POINT)
      QX=X4*12.
      QY=Y4*12.
      QP=P4/144.
      WRITE(6,1006)QX,QY,XM4,TH44,T4,QP
      WRITE(6,206)
  206 FORMAT(1H0,12HCORNER POINT)
      QX=X(K)*12.
      QY=Y(K)*12.
      QP=P3/144.
      WRITE(6,1006)QX,QY,XMP3,THP,T3,QP
      VLP(M)=V3
      XP(M)=X(K)
```

```
      YP(M)=Y(K)
      XMP(M)=XMP3
      TH(M)=TH3
      TP(M)=T3
      PP(M)=P3
      GO TO (51,52),KEY
 51   J=666
      RETURN
 65   TH3=TH(N)
      XMP3=XMP(N)
      NZ=N
      P3=PP(N)
      T3=TP(N)
      K=NX
      X(K)=XP(N)
      Y(K)=YP(N)
 52   READ(5,1007)NU,PA,KKD
      IF(KKD.EQ.0)GO TO 401
      PA=PA*144.
 401  WRITE(6,207)
```

LIST OF FORTRAN PROGRAM

PLUG NOZZLE ANALYSIS BY USING THE METHOD OF CHARACTERISTICS

```
 207 FORMAT(1H0,29HRIGHT RUNNING CHARACTERISTICS)
1007 FORMAT(I2,E15.8,I2)
     C3=(GAM+1.)/GM1
     C4=1./C3
     TERM=TH3-SQRT(C3)*ATAN(SQRT(C4*(XMP3**2-1.)))+ATAN(SQRT(XMP3**2-1.
    1))
     XME=SQRT(2./GM1*((1.+GM1/2.*XMP3**2)/((PA/P3)**(GM1/GAM))-1.)
    1)
     XNU=NU
     DM=(XME-XMP3)/(XNU-1.)
     XM=XMP3
  53 DO 54 II=1,NU
     RXM(II)=XM
     RTH(II)=TERM+SQRT(C3)*ATAN(SQRT(C4*(XM*XM-1.)))-ATAN(SQRT(XM*XM-1.
    1))
     RTP(II)=T3/(1.+GM1/2.*XM*XM)*(1.+GM1/2.*XMP3**2)
     RPP(II)=P3*((1.+GM1/2.*XMP3**2)/(1.+GM1/2.*XM*XM))
    1**(GAM/GM1)
     RTHP=RTH(II)*57.29578
     XM=XM+DM
```

LIST OF FORTRAN PROGRAM

PLUG NOZZLE ANALYSIS BY USING THE METHOD OF CHARACTERISTICS

```
      YB1=Y(J)
      YB2=Y(K)
      XB=XB1
      BB=B1
      VB=V1
      TB=TP(L)
      AB=A1
      KKNT=0
   11 KKNT=KKNT+1
      KCNT=0
      IF(KKNT-50)111,111,133
  133 WRITE(6,134)XBP,XB,XB1,XB2
  134 FORMAT(1H0,4E15.8)
      GO TO 13
  111 TH(M)=ATAN((YB2-YB1)/(XB2-XB1))
   22 GO TO (33,44),KODE
   33 Z1=(TH(L)-B1+TH(M)-BB)/2.
      Z9=TH(L)-TH(M)
      GO TO 55
   44 Z1=(TH(L)+B1+TH(M)+BB)/2.
```

LIST OF FORTRAN PROGRAM

PLUG NOZZLE ANALYSIS BY USING THE METHOD OF CHARACTERISTICS

```
      Z9=TH(M)-TH(L)
      GO TO 55
   55 Z2=COS(Z1)
      KCNT=KCNT+1
      Z10=SIN((B1+BB)/2.)
      Z3=SIN(Z1)/Z2
      Z4=SIN(TH(M))/COS(TH(M))
      Z5=SIN((TH(L)+TH(M))/2.)
      Z7=(B1+BB)/2.
      Z8=COS(Z7)
      Z6=SIN(Z7)
      XBP=(YP(L)-YB1-XP(L)*Z3+XB1*Z4)/(Z4-Z3)
      YP(M)=YB1+(XBP-XB1)*Z4
      VBP=V1+((V1+VB)/2.*Z6/Z8)*(Z9+(Z5*Z10/((YP(L)+YP(M))/2.*Z2)))*(
     1XBP-XP(L))-((TP(L)+TB)/2./(((A1+AB)/2.)**2)*Z6*Z8)*(SB-S1)*G
   12 HBP=H-VBP*VBP/2.
      AB =SQRT(GM1*HBP)
      XMP(M)=VBP/AB
      TB =GM1*HBP/(GAM*R*G)
      IF(XMP(M)-1.)997,998,998
```

LIST OF FORTRAN PROGRAM

PLUG NOZZLE ANALYSIS BY USING THE METHOD OF CHARACTERISTICS

```
997 WRITE(6,1006)XMP(M)
    STOP
998 CONTINUE
    IF(ABS((XBP-XB)/XB)-.000001)3,3,4
  4 BB=ARSIN(1./XMP(M))
    XB=XBP
    ITR=ITR+1
    VB=VBP
    IF(KCNT-50)22,22,333
333 WRITE(6,134)XBP,XB,XB1,XB2
  3 IF(XB1-XBP)6,13,5
  6 IF(XBP-XB2)13,13,9
  9 XB1=XB2
    YB1=YB2
    J=J+1
    K=K+1
    IF(K-NX)21,21,200
200 TH(M)=XTH3
    YP(M)=XY3
    XMP(M)=XXMX3
```

LIST OF FORTRAN PROGRAM

PLUG NOZZLE ANALYSIS BY USING THE METHOD OF CHARACTERISTICS

```
      GO TO 20
   21 XB2=X(K)
      YB2=Y(K)
      GO TO 11
    5 XB2=XB1
      YB2=YB1
      J=J-1
      K=K-1
      XB1=X(J)
      YB1=Y(J)
      IF(J)20,20,11
   13 THB2=TH(M)*57.29578
      PP(M)=PP(M)*(TB/TP(M))**(GAM/GM1)
      TP(M)=TB
      QX=XBP*12.
      QY=YP(M)*12.
      QP=PP(M)/144.
      WRITE(6,1006)QX,QY,XMP(M),THB2,TP(M),QP,ITR
      VLP(M)=VBP
      XP(M)=XBP
```

```
      RETURN
 1006 FORMAT(1H0,6(3X,E15.8),I5)
      END
      SUBROUTINE FLDRTN (IZ)
      DIMENSION YP(400),XP(400),TH(400),XMP(400),TP(400),PP(400),
     1RXM(200),RTH(200),RTP(200),RPP(200),VLP(400)
      DIMENSION H(3),A(3),V(3),B(3),S(3)
      COMMON YP,XP,TH,XMP,TP,PP,RXM,RTH,RTP,RPP,VLP,ROS,YS,GAM,GM1,G,
     1XM,N,P,T,R,L,M,J,N2,XXB2,YYB2,NU,KNT,GP1,FE,RT
     1,PA
      WRITE(6,2)
    2 FORMAT(1H0,40X,13HFIELD ROUTINE)
      MS=1
      CP=GAM*R/GM1
      GO TO (23,23,24),IZ
   23 II=L
      GO TO 25
   24 II=M
   25 DO 10 IJ=L,M
      ITR=1
```

CS PLUG NOZZLE ANALYSIS BY USING THE METHOD OF CHARACTERISTICS

```
      GO TO (32,32,18),IZ
   18 J=J-1
      GO TO 11
   32 J=J+1
      GO TO 33
   33 GO TO (11,12),IZ
   12 IF(II-M)11,13,13
   13 IF(KNT-NU)14,14,11
   14 MS=2
      ST=TP(J+1)
      SP=PP(J+1)
      SM=XMP(J+1)
      SH=TH(J+1)
      SX=XP(J+1)
      SY=YP(J+1)
      TP(J+1)=RTP(KNT)
      PP(J+1)=RPP(KNT)
      XMP(J+1)=RXM(KNT)
      TH(J+1)=RTH(KNT)
      XP(J+1)=XXB2
```

LIST OF FORTRAN PROGRAM

PLUG NOZZLE ANALYSIS BY USING THE METHOD OF CHARACTERISTICS

```
      YP(J+1)=YYB2
   11 DO 8 I=1,2
      GO TO (19,19,20),IZ
   19 J1=J+I-1
      GO TO 21
   20 J1=J+2*I-2
   21 H(I)=CP*TP(J1)*G
      A(I)=SQRT(GM1*H(I))
      V(I)=XMP(J1)*A(I)
      B(I)=ARSIN(A(I)/V(I))
      PT=P*(2./GP1)**(GAM/GM1)
      TT=T*(2./GP1)
      S(I)=CP*ALOG((TP(J1)/TT)/((PP(J1)/PT)**(GM1/GAM)))
    8 CONTINUE
      W=H(1)+V(1)*V(1)/2.
      TP(II)=TP(J)
      B(3)=B(1)
      TH(II)=TH(J)
      S(3)=S(1)
      A(3)=A(1)
```

LIST OF FORTRAN PROGRAM

PLUG NOZZLE ANALYSIS BY USING THE METHOD OF CHARACTERISTICS

```
      V(3)=V(1)
    4 GO TO (26,26,27),IZ
   26 JP=J+1
      GO TO 28
   27 JP=J+2
   28 Z1=(TH(J)+B(1)+TH(II)+B(3))/2.
      Z2=(TH(JP)-B(2)+TH(II)-B(3))/2.
      Z4=(B(1)+B(3))/2.
      Z5=(B(2)+B(3))/2.
      Z6=(V(1)+V(3))/2.
      Z7=(V(2)+V(3))/2.
      Z12=COS(Z1)
      Z13=COS(Z2)
      Z16=COS(Z4)
      Z17=COS(Z5)
    5 FORMAT(1H0,6(3X,E15.8),I5)
      Z8=SIN(Z1)/Z12
      Z9=SIN(Z2)/Z13
      Z10=SIN(Z4)
      Z11=SIN(Z5)
```

PLUG NOZZLE ANALYSIS BY USING THE METHOD OF CHARACTERISTICS

```
 Z14=Z16/Z10
 Z15=Z17/Z11
 Z18=(TP(J)+TP(II))/2.
 Z19=(TH(J)+TH(II))/2.
 Z20=(TH(JP)+TH(II))/2.
 Z21=2.*Z18
 Z22=((A(1)+A(3))/2.)**2
 XP(II)=(XP(J)+1./Z8*(YP(JP)-YP(J)-XP(JP)*Z9))/(1.-Z9/Z8)
 Z25=XP(II)-XP(JP)
 Z26=XP(II)-XP(J)
 YP(II)=YP(JP)+Z9*Z25
 Z23=(YP(J)+YP(II))/2.
 Z24=(YP(JP)+YP(II))/2.
 S(3)=S(1)+((S(2)-S(1))*Z26*(Z10/Z12))/(Z26*Z10/Z12+Z25*Z11/Z13)
 V(3)=1./(Z14/Z6+Z15/Z7)*(TH(JP)-TH(J)+Z14/Z6*V(1)+Z15/Z7*V(2)+
1Z10*SIN(Z19)/(Z23*Z12)*Z26+Z11*SIN(Z20)/(Z24*Z13)*Z25-Z18/(Z22)*Z
210*Z16*(S(3)-S(1))*G-(Z18/Z22)*Z11*Z17*(S(3)-S(2))*G)
 TH3P=TH(J)+(V(3)-V(1))*(Z14/Z6)-(Z10*SIN(Z19))/(Z23*Z12)*Z26+Z18/
1Z22*Z10*Z16*(S(3)-S(1))*G
 H(3)=W-V(3)*V(3)/2.
```

LIST OF FORTRAN PROGRAM

PLUG NOZZLE ANALYSIS BY USING THE METHOD OF CHARACTERISTICS

```
      A(3)=SQRT(GM1*H(3))
      TP(II)=GM1*H(3)/(GAM*R*G)
      IF(ITR-50)67,67,68
   68 WRITE(6,5)TH3P,TH(II)
      GO TO 6
   67 IF(ITR-1)7,7,66
   66 IF(ABS(TH3P-TH(II))-.000001)6,6,7
    7 B(3)=ARSIN(A(3)/V(3))
      TH(II)=TH3P
      ITR=ITR+1
      GO TO 4
    6 TH(II)=TH3P
      VLP(II)=V(3)
      THPP=TH(II)*57.29578
      PP(II)=PP(J)*((H(3)/H(1))**(GAM/GM1))/(EXP((S(3)-S(1))/R))
      XMP(II)=V(3)/A(3)
      QX=XP(II)*12.
      QY=YP(II)*12.
      QP=PP(II)/144.
      WRITE(6,5)QX,QY,XMP(II),THPP,TP(II),QP,ITR
```

LIST OF FORTRAN PROGRAM

PLUG NOZZLE ANALYSIS BY USING THE METHOD OF CHARACTERISTICS

```
      GO TO (30,30,31),IZ
   30 II=II+1
      GO TO 10
   31 II=II-1
   10 CONTINUE
      GO TO (17,16),MS
   16 TP(J+1)=ST
      PP(J+1)=SP
      XMP(J+1)=SM
      TH(J+1)=SH
      XP(J+1)=SX
      YP(J+1)=SY
      KNT=KNT+1
   17 RETURN
      END
      SUBROUTINE BPRS
      DIMENSION YP(400),XP(400),TH(400),XMP(400),TP(400),PP(400),
     1RXM(200),RTH(200),RTP(200),RPP(200),VLP(400)
      DIMENSION ZC(100),ZJ(100),XM1(100),PBP1(100)
      COMMON YP,XP,TH,XMP,TP,PP,RXM,RTH,RTP,RPP,VLP,ROS,YS,GAM,GM1,G,
```

LIST OF FORTRAN PROGRAM

PLUG NOZZLE ANALYSIS BY USING THE METHOD OF CHARACTERISTICS

```
     1XM,N,P,T,R,L,M,J,N2,XXB2,YYB2,NU,KNT,GP1,FE,RT
     1,PA
      COMMONZC,ZJ,XM1,PBP1,NQ
      DM=.1
      K=1
      XM2=1.5
    2 CZ=XM2**2/(2./GM1+XM2**2)
      QJ=TABLE1(ZJ,ZC,CZ,NQ)
      CD=QJ*QJ*CZ
      ZZ=1./((1.-CD)**(GAM/GM1))
      TZZ=((5.*(ZZ-1.))/(7.*XM2**2-5.*(ZZ-1.)))**2*((7.*XM2**2-(6.*ZZ+1.
     1))/(6.*ZZ+1.))
      T34=ATAN(SQRT(TZZ))
      W1=SQRT(GP1/GM1)*ATAN(SQRT(GM1/GP1*(XM2**2-1.)))-ATAN(SQRT(XM2
     1**2-1.))-T34
      CALL CONVR(2,XM,W1)
      P1PO=(1.+GM1/2.*XM**2)
      POP2=(1.+GM1/2.*XM2**2)
      PBP1(K)=(P1PO/POP2)**(GAM/GM1)
      XM1(K)=XM
```

LIST OF FORTRAN PROGRAM

PLUG NOZZLE ANALYSIS BY USING THE METHOD OF CHARACTERISTICS

```
      XM2=XM2+DM
      K=K+1
      IF(K-100)7,7,1
    7 IF(XM-6.)2,1,1
    1 NQ=K-1
      RETURN
      END
      SUBROUTINE CONVR(KODE,XM,ANGLE)
      DIMENSION YP(400),XP(400),TH(400),XMP(400),TP(400),PP(400),
     1RXM(200),RTH(200),RTP(200),RPP(200),VLP(400)
      COMMON YP,XP,TH,XMP,TP,PP,RXM,RTH,RTP,RPP,VLP,ROS,YS,GAM,GM1,G,
     1XZ,N,P,T,R,L,M,J,N2,XXB2,YYB2,NU,KNT,GP1,FE,RT
     1,PA
      GO TO (1,2),KODE
C     KODE=1--INPUT M, COMPUTE ANGLE
C     KODE=2--INPUT ANGLE,COMPUTE M
    1 ANGLE=SQRT(GP1/GM1)*ATAN(SQRT((GM1/GP1)*(XM*XM-1.)))
     1-ATAN(SQRT(XM*XM-1.))
      RETURN
    2 XM=10.
```

LIST OF FORTRAN PROGRAM

PLUG NOZZLE ANALYSIS BY USING THE METHOD OF CHARACTERISTICS

```
      J=0
      KEY=0
      DXM=1.
   55 IF(J-50)5,13,13
    5 FMI=SQRT(GP1/GM1)*ATAN(SQRT(GM1/GP1*(XM*XM-1.)))-ATAN(SQRT(XM*XM
     1-1.))
      TEST=FMI-ANGLE
      IF(KEY)4,4,3
    4 XM=XM-DXM
      IF(TEST)8,13,9
    9 KEY=1
      GO TO 5
    8 KEY=2
      GO TO 5
    3 GO TO (6,7),KEY
    6 IF(TEST)10,13,11
   11 XM=XM-DXM
      J=J+1
      IF(ABS(TEST)-.000001)13,13,55
   10 XM=XM+DXM
```

```
      DXM=DXM/10.
      GO TO 11
    7 IF(TEST)11,13,12
   12 XM=XM+DXM
      DXM=DXM/10.
      GO TO 11
   13 RETURN
      END
      FUNCTION TABLE1(F1,F2,F3,NPTS)
      DIMENSION F1(100),F2(100)
      IF(F2(1)-F2(NPTS))230,230,235
  235 DO 240 K=1,NPTS
      I=K
      IF(F2(I)-F3)30,20,240
  240 CONTINUE
  230 DO 10 K=1,NPTS
      I=K
      IF(F2(I)-F3)10,20,30
   10 CONTINUE
   20 TABLE1=F1(I)
```

```
      GO TO 40
   30 IF(I-1)1,1,2
    1 TABLE1=F1(I)
      GO TO 40
    2 A1=F2(I-1)
      A2=F1(I-1)
    3 TABLE1=(F1(I)-A2)*(F3-A1)/(F2(I)-A1)+A2
   40 CONTINUE
      RETURN
      END
END-OF-DATA ENCOUNTERED ON SYSTEM INPUT FILE.
```

LIST OF FORTRAN PROGRAM

PLUG NOZZLE ANALYSIS BY USING THE METHOD OF CHARACTERISTICS

```
      J=0
      KEY=0
      DXM=1.
   55 IF(J-50)5,13,13
    5 FMI=SQRT(GP1/GM1)*ATAN(SQRT(GM1/GP1*(XM*XM-1.)))-ATAN(SQRT(XM*XM
     1-1.))
      TEST=FMI-ANGLE
      IF(KEY)4,4,3
    4 XM=XM-DXM
      IF(TEST)8,13,9
    9 KEY=1
      GO TO 5
    8 KEY=2
      GO TO 5
    3 GO TO (6,7),KEY
    6 IF(TEST)10,13,11
   11 XM=XM-DXM
      J=J+1
      IF(ABS(TEST)-.000001)13,13,55
   10 XM=XM+DXM
```

LIST OF FORTRAN PROGRAM

PLUG NOZZLE ANALYSIS BY USING THE METHOD OF CHARACTERISTICS

```
      DXM=DXM/10.
      GO TO 11
    7 IF(TEST)11,13,12
   12 XM=XM+DXM
      DXM=DXM/10.
      GO TO 11
   13 RETURN
      END
      FUNCTION TABLE1(F1,F2,F3,NPTS)
      DIMENSION F1(100),F2(100)
      IF(F2(1)-F2(NPTS))230,230,235
  235 DO 240 K=1,NPTS
      I=K
      IF(F2(I)-F3)30,20,240
  240 CONTINUE
  230 DO 10 K=1,NPTS
      I=K
      IF(F2(I)-F3)10,20,30
   10 CONTINUE
   20 TABLE1=F1(I)
```

LIST OF FORTRAN PROGRAM

PLUG NOZZLE ANALYSIS BY USING THE METHOD OF CHARACTERISTICS

```
      GO TO 40
   30 IF(I-1)1,1,2
    1 TABLE1=F1(I)
      GO TO 40
    2 A1=F2(I-1)
      A2=F1(I-1)
    3 TABLE1=(F1(I)-A2)*(F3-A1)/(F2(I)-A1)+A2
   40 CONTINUE
      RETURN
      END
END-OF-DATA ENCOUNTERED ON SYSTEM INPUT FILE.
```